TORQUE

* causes rotation

* created by a Force applied in a certain way

$$\vec{\tau} = \vec{r} \times \vec{F}$$

torque — $\vec{\tau}$

position vector — $\vec{r}$

Force — $\vec{F}$

units: Nm

NOT energy!

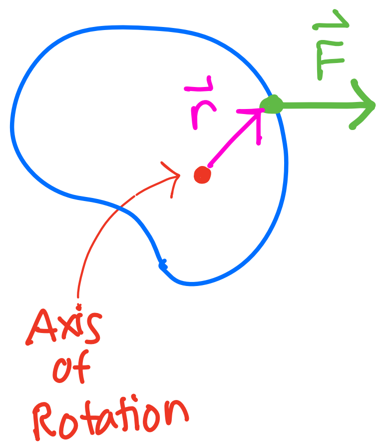
Remember that a cross product is a vector. We will deal with the direction of torque later. To find the magnitude of the torque it is best to think of it as

$$\tau = r F_{\perp}$$

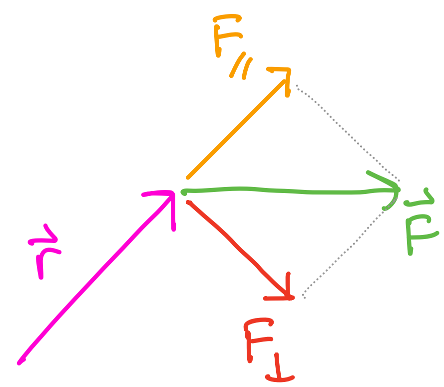
← magnitude!

When we did Newton's Second Law ($\Sigma \vec{F} = m\vec{a}$) we used Free Body Diagrams - and treated ALL the forces as if they acted on the center of mass. That was correct because $\Sigma \vec{F} = m\vec{a}$ only deals "translation" - the motion of the center of mass.

When dealing with rotations and torques however, we now need to look at exactly where a force is "touching" the object in order to determine the resulting torque.



The picture shows a random object that can rotate about the axis shown. A force $\vec{F}$ is exerted on the object, and the vector $\vec{r}$ points from the axis of rotation, to where the force is touching the object.



$\vec{r}$ and $\vec{F}$ are redrawn to the left. Notice how part of $\vec{F}$ is perpendicular to $\vec{r}$ ($F_{\perp}$) and part is parallel to $\vec{r}$ ($F_{\parallel}$).

The torque causing the rotation comes from the perpendicular part only.

In order for a force to create a torque, two things must be true.

1. The force must <u>NOT</u> be exerted on the axis of rotation. $(r=0)$

   $\vec{r}$ is the vector that points from the axis of rotation to where the force is touching the object.* see note below

2. the Force must <u>NOT</u> be parallel to $\vec{r}$.

   Only the part of the force that's $\perp$ to $\vec{r}$ helps to create the torque.

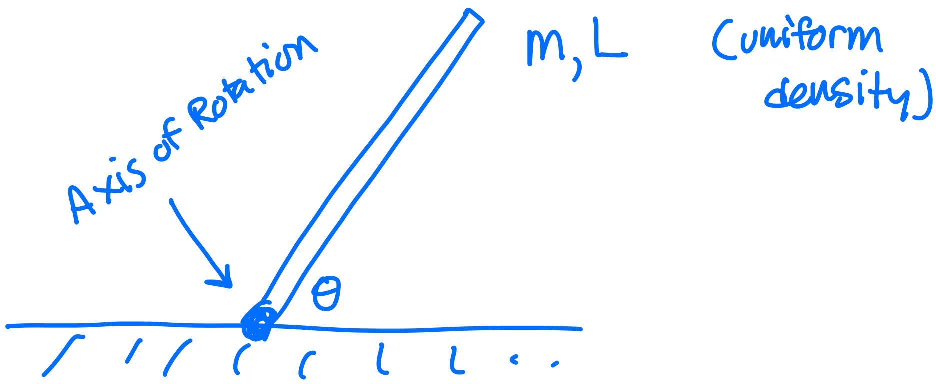
$$\vec{\tau} = \vec{r} \times \vec{F} \quad \& \quad \tau = rF_{\perp}$$

* Note: The force of gravity acts on the object's center of mass, even though nothing is "touching" the object for gravity.

No rotation, $\Sigma \tau = 0$.

[just like something @ rest $\Sigma F = 0$]

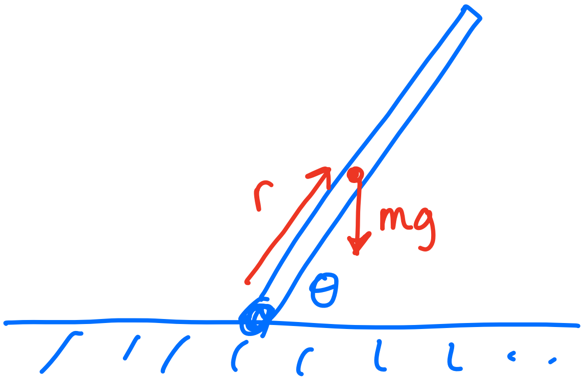
Example: Find the magnitude of the torque created by the weight of a pole that is leaning. Pole is mass "m" and length "L" and is free to rotate about its end point on the ground. The angle "θ" is shown.

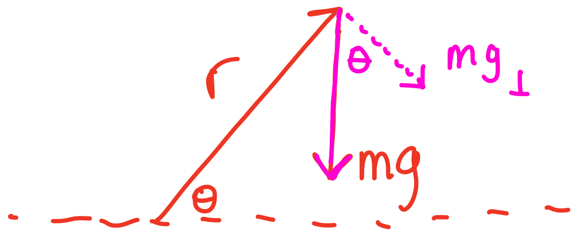


So $\vec{\tau} = \vec{r} \times \vec{F}$ and $\tau = r F_\perp$. We need to figure out $\vec{r}$, $\vec{F}$ (and then $F_\perp$.)

For figuring out torques, we treat gravity as if it is acting on the center of mass - which will be the center of the pole. Therefore, $\vec{r}$ points from the axis to the center of the pole, so $r = \frac{L}{2}$.

$\vec{F}$ is just mg, directed down - to find $F_\perp$ we need to look at the geometry:





So $\underline{\underline{r = L/2}}$ and $\underline{\underline{F_\perp = mg\cos\theta}}$

$$\tau = rF_\perp$$

$$\tau = \frac{L}{2} mg\cos\theta$$